DOCUMENT RESUME

ED 448 209 TM 032 239

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TITLE Alternative Approaches for Interpreting Alpha with Homogeneous Subsamples.
PUB DATE 2000-11-16
NOTE 20p.; Paper presented at the Annual Meeting of the Mid-South Educational Research Association (28th, Bowling Green, KY, November 17-19, 2000).
PUB TYPE Opinion Papers (120) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Estimation (Mathematics); *Reliability; *Researchers
IDENTIFIERS *Alpha Coefficient; *Homogeneity Tests

ABSTRACT

Much of the current research concerning reliability emphatically suggests that researchers should gather their own reliability estimates when administering an instrument. It has also been recommended that data with low reliability be discarded. While some data obtained from instruments that originally yielded reliable results may be unreliable, it does not necessarily follow that the data are not useful to researchers. This paper contends that although data that are homogeneous might yield less reliable results than an inducted sample, these data should not be discarded until further examination of the data is conducted. Methods for determining data homogeneity are discussed in detail. The examination of the mean item variances, the variance of the mean item variances, and the squared standard error of measurement is encouraged, but, in fact, no specific guidelines have been developed for determining what is and is not an acceptable threshold for data homogeneity. Contains 3 tables and 11 references.) (Author/SLD)

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Alternative Approaches for Interpreting Alpha with Homogeneous Subsamples

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TM032239

Paper presented at the annual meeting of the Mid-South Educational Research Association, Bowling Green, KY, November 16, 2000.
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Abstract

Much of the current research concerning reliability is emphatically suggesting that researchers gather their own reliability estimates when administering an instrument. It has also been recommended that data with low reliability then be discarded. While some data obtained from instruments that originally yielded reliable results may be unreliable, it does not necessarily follow that the data are unuseful to researchers. This paper will contend that although data that are homogeneous might yield less reliable results than an inducted sample, these data should not be discarded until further examination of the data is conducted. Methods for determining data homogeneity will be discussed in detail.

Alternative Approaches for Interpreting Alpha with Homogeneous Subsamples

Much of the current literature concerning reliability is emphatically suggesting that researchers obtain their own reliability estimates when gathering new data from a previously-developed instrument (Vacha-Haase, Kogan, & Thompson, in press). Moreover, Thompson and Vacha-Haase (2000), encouraged researchers not to "induct" reliability estimates for their own dataset from previous studies, but to obtain reliability estimates for their own dataset because reliability estimates are affected by individual sample characteristics. Concerning this practice of "inducting" reliability estimates, Pedhazur and Schmelkin (1991) stated:

> [Reliability estimates printed in test manuals] may be useful for comparative purposes, but it is imperative to recognize that the relevant reliability estimate is the one obtained for the sample used in the [current] study under consideration. (p. 86)

Similarly, Dawis (1987) contended that although the type of instrument utilized may influence reliability, reliability also might be influenced by sample composition and sample variability. Therefore, it is imperative that researchers compute and interpret reliability coefficients for their underlying sample.

Noting the problems with interpreting alpha as a consistent measure of reliability, Pedhazur and Schmelkin (1991) stated, "coefficient alpha will underestimate the reliability of a measure when its items are not at least essentially tau-equivalent" (p. 100). Pedhazur and Schmelkin (1991) then give a brief discussion of the problems associated with interpreting alpha with differing test lengths (number of items), with restricting time allotted to administer the test,

and when instrument homogeneity (unidimensionality) is high. The last of these problems, homogeneity, is the focus of the present essay.

Coefficient Alpha

For the purposes of this paper, we will use the formula for coefficient alpha that was originally developed by Cronbach (1951). For reference purposes, the formula for computing alpha is

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\Sigma\sigma_i^2}{\sigma_x^2}\right) \qquad (1)$$

where k is the number of items, $\Sigma\sigma_i^2$ is the sum of the individual test item variances, and σ_x^2 is the total test variance. Although typically KR-20 is used with dichotomously-scored items, it should be noted that alpha equals KR-20 because each kth $p_k q_k$ equals each ith σ_i^2, and across all k items $\Sigma p_k q_k = \Sigma\sigma_i^2$ (Thompson, 1999). Consulting Equation 1, a large alpha would be yielded from scores that necessarily had a small sum of individual item variances and a large total test variance. Likewise, a dataset yielding a small alpha coefficient would be produced by scores that had high individual item variances and a small total test variance. It should also be noted that although conceptually alpha represents a squared metric, it is mathematically possible to obtain an alpha value less than zero. This occurs when the sum of the individual item variances exceeds the total test variance.

Consider the following heuristic example of datasets that would yield a low reliability estimate. The first example is a measure given in which all of the students simply guessed at responses to items and the variance of the individual items was as large or even larger than the total test variance. This might be the case if an instrument like the Scholastic Assessment Test was given to second graders. The second example is when there is a lack of variability among

examinees. For example, if a second-grade spelling test was administered to college English majors, all of the examinees would probably score close to the same value, thus yielding a small alpha.

It has been recommended consistently in the literature that scores yielding low reliability estimates either be considered extremely suspect or discarded (Abelson, 1997). While in many instances, scores that yield low reliability coefficients indicate poor psychometric properties, it does not necessarily follow that the underlying data are always "unuseful" to researchers. Consider an example when a depression measure is administered to students who are all relatively not depressed, that is, who represent a homogeneous sample with respect to depression scores. Such scores likely would yield a low reliability estimate. However, interpreting this reliability coefficient without taking into consideration the homogeneous nature of the sample would be misleading. Rather, what is needed in such instances is not to discard the data, but to examine the scores further to determine why low reliability estimates were obtained from an instrument that produced high reliability estimates in the original test sample. In an attempt to overcome problems associated with low reliability estimates in homogeneous samples, Magnusson (1967) offered a formula based on the sample variance from the original normative group (i.e., the inducted sample) and the underlying sample group. Magnusson's (1967) formula can be used to predict the reliability of scores from a present sample, based on the reliability of scores from the inducted sample and the standard deviations of both samples.

Heuristic Example

For the purposes of the present article, two heuristic datasets were utilized that were both (hypothetically) derived from the same eight-item instrument. The first dataset was designed such that the scores reflected what a homogeneous sample might look like if drawn from a

population of scores. It should be noted that there are slight variations within each item because including data in which all of the examinees had the same score contributes nothing to the variability of the data. The second dataset was generated to represent a completely random set of scores. The data used are illustrated in Tables 1 and 2. (We would like to bring to the reader's attention the fact that we are not advocating that such small samples be utilized, due to their ensuing low statistical power for detecting relationships. These small datasets are used for illustrative purposes only.)

Insert Tables 1 and 2 about here

Once data were generated, reliability analyses were conducted using the Statistical Package for the Social Sciences (SPSS; SPSS Inc., 2000). Results from the reliability analyses are provided in Table 3. The second column in Table 3 illustrates the variance and reliability estimates for the random data in Table 2. The third column represents the variance and reliability estimates for the homogeneous data in Table 1. In the fourth column, a hypothetical dataset also has been included to simulate plausible test manual data. We will use this dataset for comparative purposes with our other two random and homogeneous datasets.

Insert Table 3 about here

From Table 3, we see that the alpha coefficients from the random, homogeneous, and inducted dataset are .097, .071, and .883, respectively. In this example, although the inducted scores yielded a high reliability coefficient, scores from the two heuristic datasets yielded low

reliability estimates. The reason that alpha on these two datasets is low is because the sum of the item variances is almost as great as the total test variance. In the inducted dataset, the sum of the item variances is small, whereas the total test variance is large, thereby yielding a large alpha coefficient.

The mean item variances in Table 3 are the overall mean of the eight item variances for each dataset. These values are .2573, .0625, and .1818 for the random, homogeneous, and inducted datasets, respectively. We see that the mean item variances effectively discriminates the random dataset from the homogeneous dataset. Specifically, whereas the mean item variance for the random dataset is larger than that for the inducted sample, the mean item variance for the homogeneous dataset is smaller than that for the inducted sample.

The variance of the mean item variances (Table 3) is simply the squared standard deviation (σ^2) of the mean item variances. These values are .0002, .0000, and .0056 for the random, homogeneous, and inducted datasets, respectively. It should be noted that a small variance of the mean item variances indicates that the mean item variances are relatively the same (e.g., either all small or all large). In each of the three datasets presented here, the variance of the mean item variances is small, thus indicating that nearly all of the item variances, within each dataset, are clustered relatively close together. Therefore, the variance of the mean item variances does not adequately discriminate the random and homogeneous samples.

Also presented in Table 3 is the ratio of the sum of the individual test item variances to the total test variance for the random, homogeneous, and inducted datasets. These values are .9132, .9376, and .2274, respectively. This ratio, which represents the last entry of the Equation 1 above, is the most important component of an alpha coefficient. Interestingly, these ratios do not adequately discriminate the random dataset from the homogeneous dataset.

The final statistic presented in Table 3 is the square of the standard error of measurement. The standard error of measurement, or the standard deviation of errors of measurement, provides an absolute rather than a relative measure of the extent to which raw and true scores are equivalent (Crocker & Algina, 1986). Although the true score can never be known, the standard error of measurement can be applied to an individual's observed score to set "plausible limits" for locating the true score. These "plausible limits" provide confidence bands for interpreting an obtained score. The smaller the standard error of measurement, the smaller the confidence band, and the greater the confidence that the observed score is near the true score. From Table 3, we see that the squared standard error of measurement effectively discriminates the random dataset from the homogeneous dataset. Specifically, whereas the squared standard error of measurement for the random dataset is larger than that for the inducted sample, the squared standard error of measurement for the homogeneous dataset is smaller than that for the inducted sample.

Detecting Homogeneous Samples: An Heuristic Example

The purpose of this heuristic example is to illustrate how researchers might identify homogeneous datasets when examining scores with low reliability estimates. A useful identifier of data homogeneity from Table 3 is the mean item variances. By comparing the mean item variance of a given sample to the mean item variance of the inducted sample, researchers can determine whether or not a given sample is more or less homogeneous than is a previous sample, based on the magnitude of difference between the two mean item variances.

Although specific rules for interpretation have not yet been developed, it would stand to reason that if the mean item variances for an observed sample was considerably smaller than the mean item variances reported in the test manual, then a researcher should examine the underlying data more closely in an effort to understand the differences in the alpha coefficients.

In particular, the difference in mean item variances can be expressed as a percentage to yield what we term a "relative mean item variance index." The relative mean item variance index is computed as

$$\frac{M\sigma_u^2 - M\sigma_{u'}^2}{M\sigma_u^2} \qquad (2)$$

where $M\sigma_u^2$ is the mean item variance for the inducted sample and $M\sigma_{u'}^2$ is the mean item variance for the underlying sample. This index ranges from $-\infty$ to 1, with positive values indicating that the study sample is more homogeneous than the inducted sample, and negative values indicating that the study sample is less homogeneous than is the original sample. This index can be expressed as a percentage by multiplying the index by 100. For the present sample, the random group has a relative mean item variance index of -.4524, whereas the index pertaining to the homogeneous dataset is .6454. The negative relative index associated with the random dataset suggests that the low reliability index is not the result of homogeneity. Conversely, the relative index value pertaining to the homogeneous dataset indicates that the low reliability coefficient obtained for the homogeneous dataset is explained to some extent by the relative homogeneous nature of the dataset. In this latter case, it could be argued that the low reliability coefficient represents a statistical artifact. As such, whereas a researcher would refrain from conducting subsequent analyses using the random dataset, use of the homogeneous dataset may be both justified and meaningful. However, it should be noted that it does not necessarily follow that if scores are identified as homogeneous then the data always should be used in an analysis. In any case, the advantage of the relative mean item variance index is that it can be compared across studies. Indeed, this information can be utilized in generalizability studies. For

example, researchers in generalizability investigations can determine how much of the variance in reliability estimates is explained by this index.

Although the variance of the mean item variances for the current example cannot be used to discriminate the random and homogeneous datasets because of their closeness in value, this statistic, alongside the mean item variances, can be utilized to eliminate a dataset from being deemed as representing a homogeneous sample. Specifically, whereas, as noted above, the mean item variances for the random dataset is relatively large, the variance of the mean item variances for the random data is very small (.0002), suggesting that each of the random data items has high variance and that the large mean item variances is not due to a specific outlier item. In a case like this, researchers would be encouraged probably to discard their data not only because the coefficient alpha is small, but also because the mean item variances suggests that the small reliability coefficient stem from a complete randomness of responses to the items.

As noted above, the squared standard error of estimate appears to be particularly useful in assessing the level of homogeneity of an underlying sample. From this estimate, we have derived a simple index, called the "relative squared standard error of estimate index," as

$$\frac{s_u^2 - s_{u'}^2}{s_u^2} \tag{3}$$

where s_u^2 is the squared standard error of estimate from inducted sample and $s_{u'}^2$ is the squared standard error of estimate from underlying sample. This index also ranges from $-\infty$ to 1, with positive values indicating that the study sample is more homogeneous than is the inducted sample, and negative values indicating that the study sample is less homogeneous than is the inducted sample. For the present heuristic example, the random group has a relative squared standard error of estimate index of -1.7140, whereas the index pertaining to the homogeneous

dataset is .3382. As for the relative mean item variance index, the negative relative squared standard error of estimate index associated with the random dataset suggests that the low reliability index is not the result of homogeneity. Conversely, the relative squared standard error of estimate index value pertaining to the homogeneous dataset indicates that the low reliability coefficient obtained for the homogeneous dataset is explained to some extent by the relative homogeneous nature of the dataset. Moreover, the positive relative squared standard error of estimate index suggests that the obtained scores are closer to the true scores than is the case for the inducted sample, providing further justification for not discarding the homogeneous dataset.

Real-Life Examples Illustrating When Homogeneity Yields Meaningful and Worthless Low Reliability Coefficients

One of the main concerns posited in this paper is that researchers, when confronted with data that yield a low alpha coefficient, should seek to investigate the reason(s) for this low reliability estimate. We believe that in the case where the data are homogeneous, it does not follow that simply discarding the data is merited. Consider the two following examples of data that would yield low reliability estimates. The first example we will refer to as an illustration of "bad" homogeneity, and the second example we will refer to as an illustration of "good" homogeneity.

After administering an examination, a researcher discovers that the dataset she is investigating had low reliability because an achievement test had been administered to a group of sixth graders that was originally intended for second graders. As a result, the entire cohort of students achieved high scores on the examination. In this instance, it could be justifiably contended that this instrument yields unreliable data for this specific sample. As has been mentioned previously, this does not mean that the instrument is unreliable (instruments are

neither reliable nor unreliable), but simply that the instrument is inappropriate for use with this cohort. This is an example of what is meant by "bad" homogeneity, or homogeneity that should consequentially lead to a discarding of the data.

The second example represents a different but equally realistic scenario. Suppose that a depression scale has been administered to 45 patients of an outpatient clinic for depression. Researchers have administered this instrument because they are concerned with the effectiveness of a certain intervention and are measuring the gain scores on the depression scale. In this case, it would be expected that the reliability estimates would be very small for the 45 patients who are relatively homogeneous because they are in a depression clinic being treated for depression. However, simply disposing with the data in this instance seems unwarranted if indeed low reliability is due to homogeneity and not to randomness. The goal of administering the instrument to these patients is to monitor the effectiveness of the intervention. In research studies like this, the effect size will be maximized if and only if participants from one extreme of the scale (e.g., clinically-depressed participants) are the focus of the study. Otherwise, ceiling effects could confound results, thereby providing rival explanations (i.e., low internal validity). Additionally, findings from such an investigation would only be generalizable (i.e., have maximal external validity) if a clinically-depressed sample is utilized. This is an example of "good" homogeneity, or homogeneity that attenuates the reliability estimate in a manner that is directly a function of the level of homogeneity of the sample.

In the second example, it is extremely likely that the mean item variances was smaller than the mean item variances reported in the test manual. If this were the case, the data should still be used in the analysis, particularly if the sample size was large, because the low reliability estimate is due to individual homogeneity and thus appears acceptable considering the context of

the study. Moreover, with respect to the homogeneous dataset presented in Table 1, it should be noted that if only the reliability coefficient in Table 3 had been reported, and further examination of the reliability properties had not been performed, readers of the subsequent final report might not have looked favorably on scores that yielded a reliability coefficient of .10.

Conclusion

Although the major focus of this paper has been to advocate that researchers spend time examining the reasons behind data yielding low reliability estimates, it should be noted that, to date, no correction statistics exist for alpha coefficients with homogeneous samples. This can and should be the focus of future research in this area. While we encourage the examination of the mean item variances, the variance of the mean item variances, and the squared standard error of measurement, no specific guidelines have been developed for determining what is and what is not an acceptable threshold for data homogeneity.

However, as illustrated above, the (squared) standard error of measurement provides extremely useful information about the degree of homogeneity of an underlying sample. This statistic has particular appeal because it is a function of both the standard deviation of scores generated from the complete test and the reliability estimate. Thus, the authors currently are attempting to develop an index that utilizes the ratio of the squared standard error of measurement for the underlying and the inducted samples.

Because low reliability estimates reduce the statistical power associated with hypothesis tests (Onwuegbuzie & Daniel, 2000), researchers should utilize larger samples, when they expect that their sample is homogeneous, in order to compensate for the corresponding reliability-based loss in statistical power. In fact, increasing the sample size (i.e., a research-based consideration) would probably provide a better correction for attenuated relationships than any statistical

correction of the reliability coefficient itself (i.e., a statistical consideration), just as randomizing participants to treatment conditions in an experimental design is superior to using analysis of covariance techniques or other statistical adjustments to analyze data stemming from non-experimental research designs.

Nevertheless, encouraging the investigation of coefficients of reliability can only help the growth of accountability in test usage. It is our contention that researchers should be encouraged not only to compute reliability estimates for their own data, but also to investigate and to explain why the reliability estimates differ from the original test manual norming (i.e., inducted) sample. Such information would allow readers to put subsequent findings in a more appropriate context.

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Table 1

Homogeneous dataset

Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
1	1	0	0	1	1	0	0
1	1	0	0	1	1	0	0
1	1	0	0	1	1	0	0
1	0	0	0	1	1	0	0
1	1	0	0	1	1	0	0
1	1	0	0	1	1	0	0
0	1	0	0	1	1	0	0
1	1	0	0	1	1	0	0
1	1	0	1	1	1	0	0
1	1	0	0	0	1	0	0
1	1	1	0	1	1	0	0
1	1	0	0	1	0	0	0
1	1	0	0	1	1	1	0
1	1	0	0	1	1	0	0
1	1	0	0	1	1	0	1
1	1	0	0	1	1	0	0

Table 2

Random dataset

Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7	Item 8
1	0	1	0	0	1	1	1
1	0	0	0	1	1	1	1
0	1	0	0	1	1	0	0
1	0	0	1	1	1	1	1
0	1	1	0	1	0	1	1
0	1	0	0	1	0	1	0
1	0	1	1	0	0	1	1
0	0	0	0	0	0	1	0
1	1	1	0	0	1	0	1
0	1	0	0	1	1	0	0
0	1	1	1	1	1	0	1
0	0	0	1	1	1	0	1
1	0	1	0	0	0	0	1
1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	1	1	0	1

Table 3

Item and Test Characteristics for the Two Heuristic Datasets and the "Inducted" Dataset

Variables	Random Dataset	Homogeneous Dataset	"Inducted" Dataset
Item 1 variance	0.2461	0.0588	0.1094
Item 2 variance	0.2461	0.0588	0.1875
Item 3 variance	0.2500	0.0588	0.2125
Item 4 variance	0.2113	0.0588	0.2148
Item 5 variance	0.2461	0.0588	0.2461
Item 6 variance	0.2461	0.0588	0.1875
Item 7 variance	0.2461	0.0588	0.0588
Item 8 variance	0.2644	0.0588	0.1094
Σ item variances	1.9262	0.4704	1.3260
Mean item variance	0.2408	0.0588	.1658
Variance of the mean item variances	0.0001	0.0000	0.0037
Total test variance	2.1094	0.5000	5.9963
Σ item variances/total test variance	0.9132	0.9408	0.2211
Alpha coefficient	0.0974	0.0714	0.8830
(Standard error of measurement)^2	2.0309	0.4952	0.7483
Relative mean item variance index	-0.4524	0.6454	
Relative squared standard error of estimate index	-1.71410	0.3382	

Note. All variance components were computed with the population size (N) and not the corrected sample size (N-1).